

Introduction to Euler Getter

Takehiko Yasuda (Osaka Univ.)

Mathematical Software and Free Documents XV

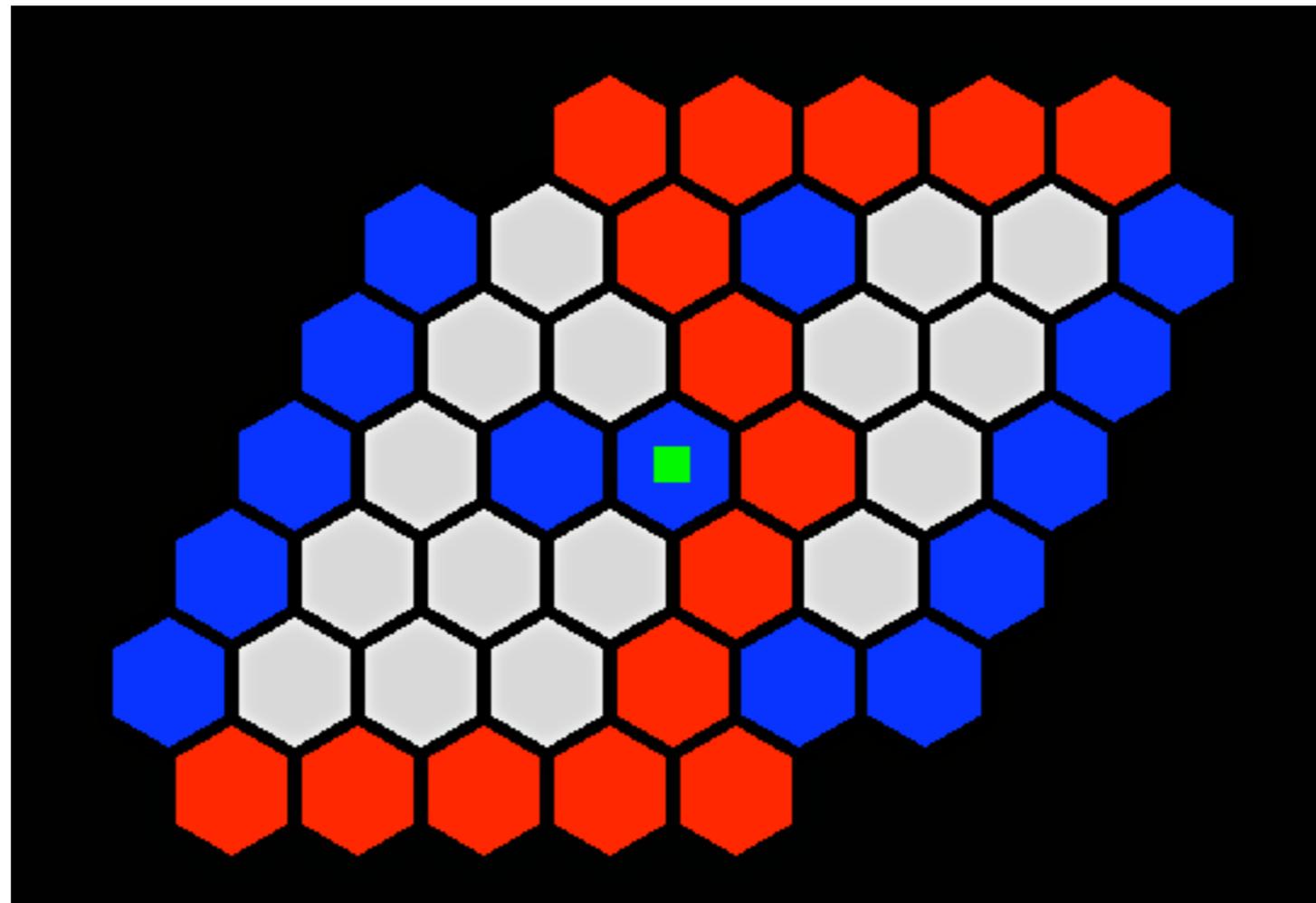
What is EG (Euler Getter) ?

a game introduced by Y. (2010)

Features	Other games with the feature
topological (connection)	Hex, Minor, Shapley
territory	Go, Reversi (Othello)

Hex

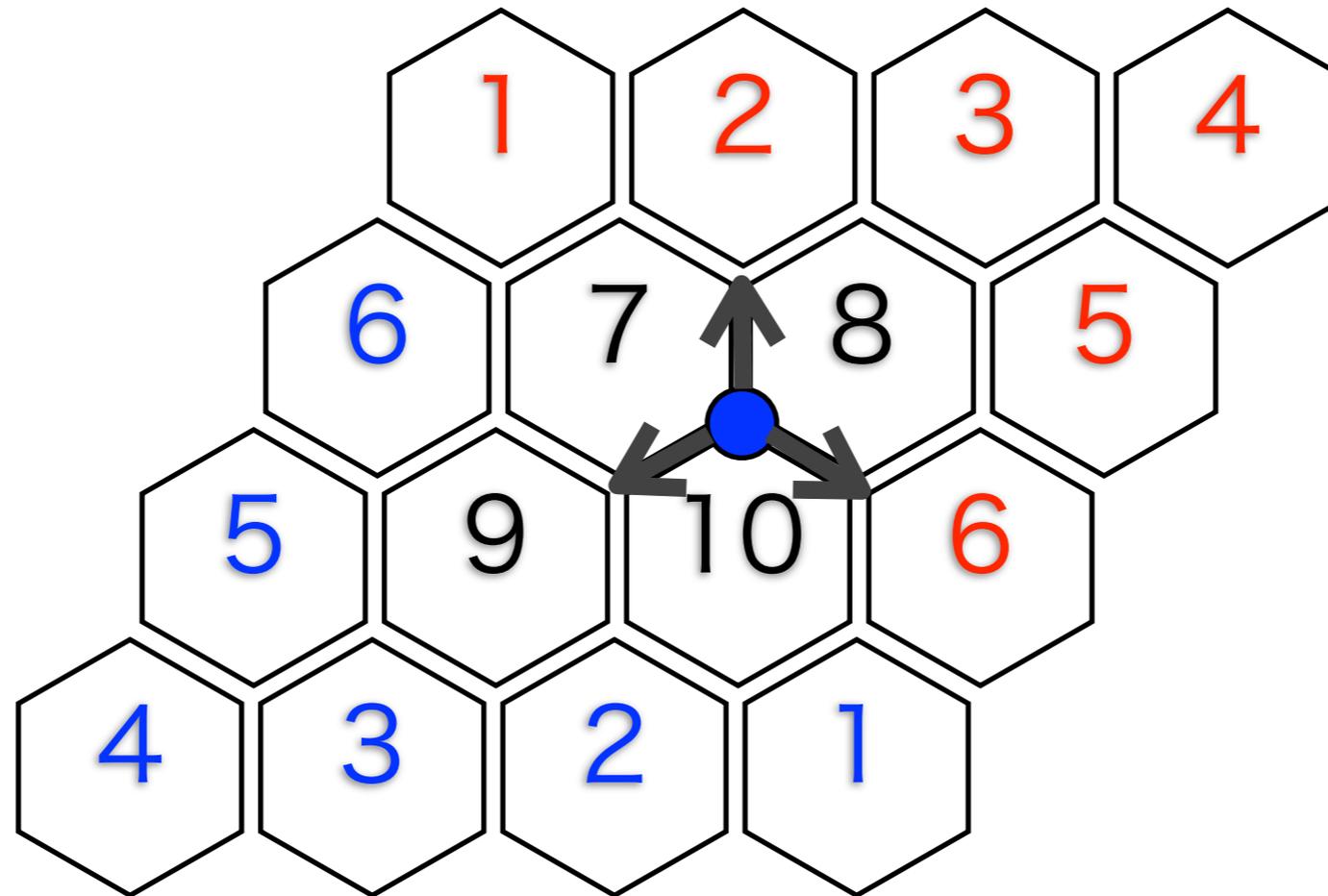
the **first topological game** invented by [Piet Hein](#)
and reinvented by [John Nash](#) (1940s)



Hex variants

- Milnor (or Y)
- Shapley (or Projective Plane, Projex)
- many others

EG Board



- real **projective plane** like Shapley
- each vertex \leftrightarrow three edges.

EG Rules

- Two players: Red and Blue
- Take turns coloring a cell red or blue, until all cells are colored.
- The winner is the one whose area has larger **Euler characteristic**.

Euler characteristic

A : area consisting of cells on a EG board

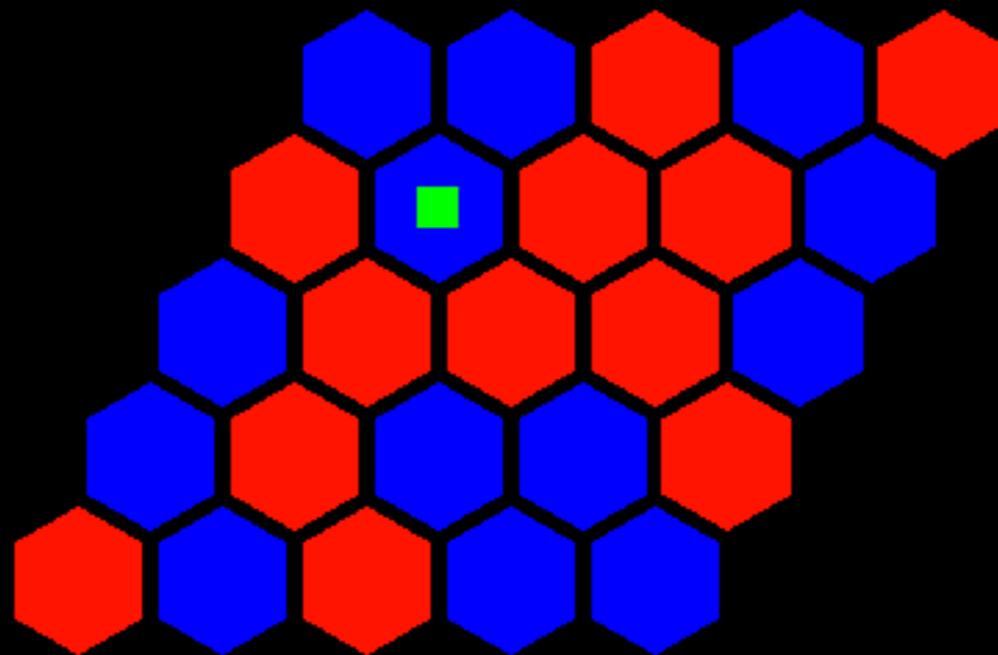
\rightsquigarrow $e(A) \in \mathbf{Z}$: its Euler characteristic

$$e(A) := \#\{\text{vertices}\} - \#\{\text{edges}\} + \#\{\text{cells}\}$$

$$= \#\{\text{connected components}\} - \#\{\text{loops}\} \leftarrow \text{human-friendly}$$

Example

$$e(A) = \#\{\text{connected components}\} - \#\{\text{loops}\}$$



Q: What are the Euler characteristics of RED and BLUE?

Euler characteristic as a measure

Inclusion-exclusion principle:

$$e(X \cup Y) = e(X) + e(Y) - e(X \cap Y)$$

The idea came from the motivic integration.

Euler characteristic as a **measure**

In EG, if the board is filled,

$$e(\text{Red}) + e(\text{Blue}) = e(\mathbf{P}^2) = 1$$

$\Rightarrow e(\text{Red}) \neq e(\text{Blue}); \text{No Draw!}$

Key Facts

- \mathbf{P}^2 is closed and unorientable.
- Red \cap Blue = disjoint loops
- $e(\text{loop}) = 0$

Winning Strategy

Theorem (Schnell)

If $\#\{\text{cells}\}$ is even, then the first player has a winning strategy.

Proof

Strategy-stealing argument

Tactics and terminology

Miura, Sannai, Shibuta, Tiba, ...

- 鋭点 (acute point)
- 鈍点 (blunt point)
- 竦み (flinch)
- 凝り (lump)
- 安息地 (haven)
- 渋田止め (Shibuta block)

Implementations

In chronological order,

- Euler Getter 1 (Y., Haskell)
- Web Euler Getter (motemen, Perl+JavaScript)
- E2G2 (Hashimoto, Maxima, **AI**)
- Euler Getter 2 (Y., Python, **AI**)
- Euler Getter 2 wrapper (Numata, Python, **AI**)

The Monte-Carlo method works well.
(Or humans are still too weak.)

Problems

- What are the best shape and size as an EG board? (Special cells like the acute point are not desirable.)
- Is the reversed rule better?
- Difficult to explain rules to the general public
- No iOS or Android implementation

S.E.G. (Stringy Euler Getter)

a possible variant of EG

which might address issues in the last slide

- on a torus instead of the projective plane
- each cell has an assigned score (randomly at the beginning)
- compete on: Euler char. + the sum of scores (+ Komi)

Algebraic-geometric interpretation

torus + scores = log elliptic curve

Euler char. + scores = stringy Euler number

References

- Euler Getter Wiki:

http://www14.atwiki.jp/euler_getter/

- My homepage:

<http://takehikoyasuda.jimdo.com/>