Recent developments from applied mathematics group of Hibi project

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Before Hibi project

- JST Hibi project started in October of 2008.
- I first briefly summarize our works on algebraic statistics before the project.
- My first manuscript on Markov basis with Satoshi Aoki was February of 2002 on $3 \times 3 \times K$ contingency tables.
- Our collaboration with Ohsugi and Hibi started around 2005.
- In April of 2007 Aoki, Hibi, Ohsugi and Takemura wrote a manuscript on group-wise selection models, which evolved into nested configurations.
- I started working with Ruriko Yoshida in 2005 and wrote some papers since then.

Before Hibi project

- Hisayuki Hara and then Tomonari Sei began producing substantial results.
- Other topics of interest involving algebraic methods:
  - Graphical models
  - Application of hyperplane arrangement theory to unfolding models (ranking models)
  - Algebraic aspects of tube formula and Euler characteristic methods (abstract tube by Naiman and Wynn is a discrete analog of the tube formula)

Objectives of applied math group

- Apply Gröbner basis technology to statistics and other areas
  - Markov basis
  - Experimental design
  - Multivariate cryptography
- Apply these techniques in practical problems
  - Analysis of large scale examination data from National Center for University Entrance Examinations (done)
  - Design of nearly optimal non-regular fractional factorial designs (this year)

Objectives of applied math group

- Collaboration with the theory group and the computation group. Statistical problems often provide challenging problems both for theory and computation.

- From the fall of last year, totally new collaboration between the computation group and the applied group started: *D*-modules are useful for computing MLE! (Takayama's talk, arXiv:1005.5273, May 2010)

Results after the start of Hibi project

- We keep writing many papers. In this aspect, the project is going well.
- Our project homepage lists 16 technical reports since October 2010.
- Some papers ((2),(6),(13)) are concerned with design of experiments, in particular fractional factorial designs.
  - Algebraic statistics approach to experimental design initiated by Pistone and Wynn suggests many interesting problems in experimental design.
  - We proposed some classes of non-regular fractional factorial designs and also studied some optimality criteria of designs.

Results after the start of Hibi project

Markov basis

- Subset of a Markov basis. Our main interest now is a subset of a Markov basis which guarantees connectivity of some fibers. We have some new results in this direction in (1) and (7). However more general results are needed.
- Markov basis for Markov chain models.
  - In two recent papers ((14),(15)) we considered Markov basis for Markov chain models.
  - Markov chain models have some interesting combinatorial features, which are reflected in their Markov bases.
  - We have explicit results for the two-state case. The case of general finite state space seems to be hard.

Results after the start of Hibi project

- Hierarchical Subspace Models for contingency tables
    - In (8) we proposed submodels of usual hierarchical models of contingency tables. The submodels preserve the conditional independence structure of the ambient hierarchical model and also allows the same localization of statistical inference.
- Hyperplane arrangement theory
    - In (11),(12) we have new developments on application of hyperplane arrangement theory to unfolding models, which are models for preference rankings.
- Moments of non-central Wishart matrix
    - In (9) we have new graph presentations of moments of elements of a noncentral Wishart matrix.
- Papers on algorithmic information theory.
    - Papers by Dr.Tadaki are concerned algorithmic information theory and randomness.